## Cambridge IGCSE ${ }^{\text {TM }}$



CENTRE NUMBER


## MATHEMATICS

0580/42
Paper 4 (Extended)
October/November 2021
2 hours 30 minutes
You must answer on the question paper.
You will need: Geometrical instruments

## INSTRUCTIONS

- Answer all questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do not use an erasable pen or correction fluid.
- Do not write on any bar codes.
- You should use a calculator where appropriate.
- You may use tracing paper.
- You must show all necessary working clearly.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.
- For $\pi$, use either your calculator value or 3.142.


## INFORMATION

- The total mark for this paper is 130 .
- The number of marks for each question or part question is shown in brackets [ ].

1 (a) Malena has 450 fruit trees.
The fruit trees are in the ratio apple : pear : plum $=8: 7: 3$.
(i) Show that Malena has 200 apple trees.
(ii) Find the number of plum trees.
(iii) Malena wants to increase the number of pear trees by $32 \%$.

Calculate the number of extra pear trees she needs.
(iv) Each apple tree produces 48.5 kg of apples.

The apples have an average mass of 165 g each.
Calculate the total number of apples produced by the 200 trees.
Give your answer correct to the nearest 1000 apples.
(b) Malena's land is valued at three million and seventy-five thousand dollars.
(i) Write this number in figures.
(ii) Write your answer to part (b)(i) in standard form.
$\qquad$
(c) In 2020, each plum tree produced 37.7 kg of plums.

This was $16 \%$ more than in 2019.
Calculate the mass of plums produced by each plum tree in 2019.
(d) Malena invests $\$ 1800$ at a rate of $2.1 \%$ per year compound interest.

Calculate the value of her investment at the end of 15 years.

2 (a) The cumulative frequency diagram shows information about the floor area, $a \mathrm{~m}^{2}$, of each of 80 houses.


Use the diagram to find an estimate of
(i) the median,
$\qquad$
(ii) the lower quartile,
$\qquad$
(iii) the interquartile range,
$\qquad$
(iv) the number of houses with a floor area greater than $120 \mathrm{~m}^{2}$.
(b) The information about the 80 floor areas is shown in this frequency table.

| Floor area <br> $\left(a \mathrm{~m}^{2}\right)$ | $40<a \leqslant 60$ | $60<a \leqslant 80$ | $80<a \leqslant 100$ | $100<a \leqslant 130$ | $130<a \leqslant 160$ | $160<a \leqslant 200$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Frequency | 14 | 17 | 18 | 15 | 9 | 7 |

(i) Calculate an estimate of the mean floor area.
$\mathrm{m}^{2}$
[4]
(ii) Complete the histogram to show the information in the frequency table.

(iii) Two of the houses are picked at random.

Find the probability that one of the houses has a floor area greater than $130 \mathrm{~m}^{2}$ and the other has a floor area $60 \mathrm{~m}^{2}$ or less.

3 (a)


NOT TO
SCALE
$A B C D E$ is a pentagon.
(i) Calculate $A D$ and show that it rounds to 94.5 m , correct to 1 decimal place.
(ii) Calculate angle $B A C$.

Angle $B A C=$
(iii) Calculate the largest angle in triangle $C A D$.
(b)


Triangle $P Q R$ has the same area as triangle $L M N$.
Calculate the shortest distance from $R$ to the line $P Q$.

(a) On the grid, draw the image of triangle $A$ after
(i) a translation by the vector $\binom{-4}{5}$,
(ii) a reflection in the line $x=1$,
(iii) an enlargement, scale factor 2 and centre $(-5,-2)$.
(b) Describe fully the single transformation that maps triangle $A$ onto triangle $B$.
$\qquad$
$\qquad$

5 The table shows some values for $y=x^{3}-3 x^{2}+3$.

| $x$ | -1 | -0.5 | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $y$ |  | 2.125 | 3 | 2.375 | 1 |  | -1 | -0.125 |  |

(a) Complete the table.
(b) On the grid, draw the graph of $y=x^{3}-3 x^{2}+3$ for $-1 \leqslant x \leqslant 3$.

[4]
(c) By drawing a suitable straight line on the grid, solve the equation $x^{3}-3 x^{2}+x+1=0$.
$\qquad$ or $x=$ $\qquad$ or $x=$

6 (a) Solve.
(i) $4(2 x-3)=24$

$$
\begin{equation*}
x= \tag{3}
\end{equation*}
$$

(ii) $6 x+14>6$
(b) Rearrange the formula $V=2 x^{3}-3 y^{3}$ to make $y$ the subject.

$$
y=
$$

(c) Show that $(2 n-5)^{2}-13$ is a multiple of 4 for all integer values of $n$.
(d) The expression $5+12 x-2 x^{2}$ can be written in the form $q-2(x+p)^{2}$.
(i) Find the value of $p$ and the value of $q$.

$$
p=. . . . . . . . . . . . . . . . ~, q=
$$

(ii) Write down the coordinates of the maximum point of the curve $y=5+12 x-2 x^{2}$.
$\qquad$
(e) The energy of a moving object is directly proportional to the square of its speed. The speed of the object is increased by $30 \%$.

Calculate the percentage increase in the energy of the object.

7 (a) The diagram shows a container for storing grain.
The container is made from a hemisphere, a cylinder and a cone, each with radius 2 m . The height of the cylinder is 5.2 m and the height of the cone is $h \mathrm{~m}$.

(i) Calculate the volume of the hemisphere.

Give your answer as a multiple of $\pi$.
[The volume, $V$, of a sphere with radius $r$ is $V=\frac{4}{3} \pi r^{3}$.]
$\qquad$ $m^{3}$
(ii) The total volume of the container is $\frac{88 \pi}{3} \mathrm{~m}^{3}$.

Calculate the value of $h$.
[The volume, $V$, of a cone with radius $r$ and height $h$ is $V=\frac{1}{3} \pi r^{2} h$.]

$$
h=
$$

(iii) The container is full of grain.

Grain is removed from the container at a rate of 35000 kg per hour. $1 \mathrm{~m}^{3}$ of grain has a mass of 620 kg .

Calculate the time taken to empty the container. Give your answer in hours and minutes.
$\qquad$ h $\qquad$ $\min [3]$
(b)


NOT TO
SCALE
$A$ and $B$ are points on a circle, centre $O$, radius $r \mathrm{~cm}$.
The area of the shaded segment is $65 \mathrm{~cm}^{2}$.
Calculate the value of $r$.

8 (a) Kaito runs along a 12 km path at an average speed of $x \mathrm{~km} / \mathrm{h}$.
(i) Write down an expression, in terms of $x$, for the number of hours he takes.
$\qquad$ hours
(ii) Yuki takes 1.5 hours longer to walk along the same path as Kaito. She walks at an average speed of $(x-4) \mathrm{km} / \mathrm{h}$.

Write down an equation, in terms of $x$, and show that it simplifies to $x^{2}-4 x-32=0$.
(iii) Solve by factorisation.

$$
x^{2}-4 x-32=0
$$

$$
x=
$$

$\qquad$ or $x=$ $\qquad$
(iv) Find the number of hours it takes Yuki to walk along the 12 km path.
$\qquad$
(b) A bus travels 440 km , correct to the nearest 10 km .

The time taken to complete the journey is 6 hours, correct to the nearest half hour.
Calculate the lower bound of the speed of the bus.

9 (a) $F$ is the point $(5,-2)$ and $\overrightarrow{F G}=\binom{-2}{3}$.
Find
(i) the coordinates of point $G$,
$\qquad$
(ii) $5 \overrightarrow{F G}$,
(iii) $|\overrightarrow{F G}|$.
(b)


NOT TO
SCALE
$O A B C$ is a parallelogram.
$P$ is a point on $A C$ and $Q$ is the midpoint of $A B$.
$\overrightarrow{O A}=\mathbf{a}$ and $\overrightarrow{O C}=\mathbf{c}$.
(i) Find, in terms of a and/or c
(a) $\overrightarrow{A Q}$,

$$
\begin{equation*}
\overrightarrow{A Q}= \tag{1}
\end{equation*}
$$

(b) $\overrightarrow{O Q}$.

$$
\overrightarrow{O Q}=
$$

(ii) $\overrightarrow{O P}=\frac{2}{3} \mathbf{a}+\frac{1}{3} \mathbf{c}$
(a) Show that $O, P$ and $Q$ lie on a straight line.
(b) Write down the ratio $O P: O Q$.

Give your answer in the form $1: n$.

10 (a) Find the coordinates of the turning points of the graph of $y=x^{3}-12 x+6$. You must show all your working.
(........... , ..........) and (.
(b) Determine whether each turning point is a maximum or a minimum. Show how you decide.

11 (a)


Triangle $P Q R$ is mathematically similar to triangle $X Y Z$.
(i) Find $Y Z$.

$$
Y Z=
$$

$\qquad$
(ii) The area of triangle $X Y Z$ is $63.6 \mathrm{~cm}^{2}$.

Calculate the area of triangle $P Q R$.
$\qquad$
(b) Two containers are mathematically similar.

The larger container has a capacity of 64.8 litres and a surface area of $0.792 \mathrm{~m}^{2}$.
The smaller container has a capacity of 37.5 litres.
Calculate the surface area of the smaller container.

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